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» Wave Motion ALPHABETICAL BIDEX ()

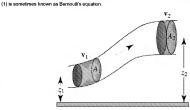
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the velocity of fluid flow (in meters per second) and h is the height above a reference surface. The second term in

this equation is known as the dynamic pressure. The effect described by this law is called the Sensoula effect, and

For a heuristic derivation of the law, picture a pipe through which and ideal fluid is flowing at a steady rate. Let W denote the work done by applying a pressure P over an area A, producing an offset of Δl , or volume change of ΔV . Let a subscript 1 denote fluid parcels at an initial point down the pipe, and a subscript 2 denote fluid parcels further down the pipe. Then the work done by pressure force

$$dW = P dV$$
 (2)

at points 1 and 2 is

$$\Delta W_i = P_1 A_1 \Delta I_i = P_1 \Delta V$$

$$\Delta W_2 = P_2 A_2 \Delta l_2 = P_2 \Delta V \qquad (4)$$

and the difference is

$$\Delta W \equiv \Delta W_1 - \Delta W_2 = P_1 \Delta V - P_2 \Delta V. \qquad (5)$$

Equating this with the change in total energy (written as the sum of knowly, and potential energies gives

$$\Delta W = \Delta K + \Delta U$$

(6)

(3)

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$$= \frac{1}{2}\Delta mv_2^2 - \frac{1}{2}\Delta mv_1^2 + \Delta mgz_2 - \Delta mgz_1.$$

Equating (6) and (5),

$$\frac{1}{2}\Delta mv_2^2 - \frac{1}{2}\Delta mv_1^2 + \Delta mgz_2 - \Delta mgz_3 = P_1\Delta V - P_2\Delta V,$$
 (7)

which, upon rearranging, gives

$$\frac{\Delta m v_2^2}{2\Delta V} + \frac{\Delta m g z_1}{\Delta V} + P_1 = \frac{\Delta m v_2^2}{2\Delta V} + \frac{\Delta m g z_2}{\Delta V} + P_2,$$
(8)

so writing the density as ho = m/V then gives

$$\frac{1}{2}\rho v^2 + \rho gz + P = [\text{const}]. \qquad (9)$$

This quantity is constant for all points along the streamline, and this is Bernoulli's theorem, first formulated by Deaded \$Arroquis \(\frac{1}{8}\) in 1738. Although it is not a new principle, it is an expression of the law of conservation of mechanical energy in a form more convenient fruit mechanical.

A more rigorous derivation proceeds using the one-dimensional Euler's equation of inviscid motion,

$$\rho u \frac{\partial u}{\partial t} = -\frac{\partial P}{\partial t}$$
(10)

along a streamline, where u is used for speed instead of v (a common convention in fluid mechanics). Integrating gives

$$\rho u du = -dP$$
 (11)

$$\frac{1}{4}pu^2 + P = [const]. \qquad (12)$$

In a gravitational field, this becomes

$$(\frac{1}{2}\rho u^2 + \rho gz + P = [\text{const}].)$$
(13)

However, if the flow has zero vorticity, then

$$\mathbf{u} \cdot \nabla \mathbf{u} = \frac{1}{2} \nabla (\mathbf{u} \cdot \mathbf{u}) - \mathbf{u} \times (\nabla \times \mathbf{u}) = \frac{1}{2} \nabla (\mathbf{u}^2),$$
 (14)

but

$$\mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla P, \tag{15}$$

so, for incompressible flow,

$$\nabla (\frac{1}{2}\rho u^2 + P) = 0 \qquad (16)$$

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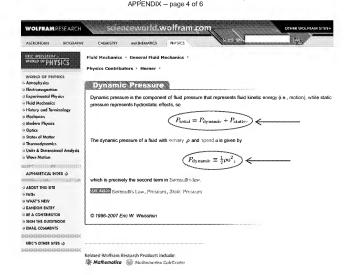
throughout the entire fluid.

Bernoulli Effect, ("Alember's Paradox, Eynamic Pressure, Kutta-Zhukovski Theorem, Ulit, Lift Coefficient, Lift Fome, Static Pressure

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Brief Overview Navier: Stokes Bernoulli Fluid Statics Glossary Flowmeters

Calculators Manager

Draining Tank
Online Flowmeter
Venturi Flowmeter
Pipe Friction
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Bernoulli Equation

A <u>non-turbulent</u>, <u>perfect</u>, <u>compressible</u>, and <u>barotropic</u> fluid undergoing steady motion is governed by the Bernoulli Equation:

$$\frac{V^2}{2g} + z + \frac{\tilde{P}}{g} = C\left(streamline\right)$$

where g is the gravity acceleration constant (9.81 m/s²; 32.2 ft/s²), V is the velocity of the fluid, and z is the height above an arbitrary datum. C remains constant along any <u>streamline</u> in the flow, but varies from streamline to streamline. If the flow is <u>irrotational</u>, then C has the same value for all streamlines.

The function p is the "pressure per density" in the fluid, and follows from the barotropic equation of state, p = p(r).

For an incompressible fluid, the function p simplifies to p/r, and the incompressible Bernoulli Equation becomes:

$$\frac{y^2}{2g} + z + \frac{p}{\rho g} = C$$

Derivation from Navier-Stokes

The Navier-Stokes equation for a perfect fluid reduce to the Euler Equation:

$$-\nabla p + \rho \mathbf{b} = \rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right)$$

Rearranging, and assuming that the body force **b** is due to gravity only, we can eventually integrate over space to remove any vector derivatives, Appl. No. Application No. 10/528,228 Response to Office Action dated Jan. 5, 2010 APPENDIX – page 6 of 6

$$\begin{split} &-\frac{1}{\rho} \vec{\nabla} p - g \mathbf{i}_{Z} = \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \vec{\nabla} \mathbf{v} \\ &-\frac{d\vec{P}}{dp} \vec{\nabla} p - \vec{\nabla} (gz) = \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \vec{\nabla} \mathbf{v} \\ &0 = \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \vec{\nabla} \mathbf{v} + \vec{\nabla} \left(\vec{P} + gz \right) \\ &C(t, streamline) = \int \frac{\partial \mathbf{v}_{t}}{\partial t} dx_{t} + \frac{1}{2}V^{2} + \vec{P} + gz \end{split}$$

If the fluid motion is also steady (implying that all derivatives with respect to time are zero), then we arrive at the Bernoulli equation after dividing out by the gravity constant (and absorbing it into the constant C),

$$\frac{V^2}{2g} + z + \frac{\tilde{P}}{g} = C \left(\text{streamline} \right)$$

Note that the fluid's barotropic nature allowed the following chain rule application,

$$\vec{\nabla} \tilde{P} = \frac{d\tilde{P}}{dp} \vec{\nabla} p = \frac{1}{\rho} \vec{\nabla} p$$

with the "pressure per density" function \tilde{p} defined as,

$$\tilde{P}(p) = \int_{p_0}^{p} \frac{d\tilde{p}}{\rho}$$

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